Remedial Flipped

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We all like to teach good students: bright, motivated, well prepared and confident. And, we get the best ones from high schools in our first year Calculus courses. Not all of them continue to be the best, but that is another story.

Should we care at all about students who, for whatever reasons, did not manage to learn the mathematics they need during their K - 12 years? Should we assume that they are not capable of ever learning it?

Once remedial, always remedial. These words were said by a professor from one of the American universities, at a workshop on transitions from high school to university. They reflect the sad truth: remedial mathematics courses offered by American colleges and universities don't work. Students repeat these courses several times, but few of them reach the level sufficient to succeed in university calculus courses. While the picture in Canada is somewhat different, there is no indication that the results are much better.

But, should we ask ourselves whether we could do a better job helping these students?

Most Canadian universities are happy to leave this problem to community colleges. Simon Fraser University is one of the exceptions. When we decided to introduce requirements for compulsory writing and quantitative courses in 2006, we also introduced two courses: FAL X99 (Foundations of Academic Literacy) and FAN X99 (Foundations of Analytic and Quantitative Reasoning), to help those students who may not be sufficiently prepared for required university courses.

Early in the process we agreed that we did not want to "go into a remedial business" – to create a sequence of several remedial courses. We wanted just one course that does it all. In addition, such a course would teach students how to think mathematically, and, hopefully, change their attitudes towards mathematics.

We are now in the eighth year of the experiment, and the course, FAN X99, has been a success. It has been evolving as we have been learning from the experience, and it will continue to evolve. But, while we cannot claim that we are reaching all students, we are making a change.

So, how does this course work?

First, unlike our regular courses, which can have up to 500 students, we teach this one in small (for us) groups of about 30 students. It would be nice to have even smaller groups, but the budget allocation limits how many sections we can have.

Second, the course employs many features of a "flipped classroom", even though we had never heard that term when creating the course.

We don't lecture, or at least try to avoid it as much as we can. Students work in groups on problems. This course inspired the introduction of hexagonal tables to some of SFU's classrooms! Theoretical concepts are introduced when the problems being worked on require them. Students are expected to complete assigned readings to learn, or review, concepts prior to the class in which they are needed.

We start the course with problems that do not rely on formulas, but on analyzing a situation, systematic investigation, or looking for a pattern. We spend a lot of time trying to understand the wording of the problems, analyzing what they say and what they ask for, and learning to talk about mathematics. And, we try to have problems that are fun – riddles or puzzle type problems are great.

There are two reasons for starting with such problems. First, each section of the course has students with very varied backgrounds – from those who have little knowledge and even less confidence in their math abilities, to those who have reasonable algebraic skills but "can't do word problems". Starting with activities that do not require specific algebraic knowledge, but at the same time are challenging for everybody, is a great equalizer.

Second, the majority of these students are used to "solving" problems by looking for an appropriate formula and substituting numbers from the problem. The only way to teach them to think is to put them in a situation where they cannot easily find a formula to use. They need to learn to make sense of a question, to look for a pattern or patterns, and to try different things. We also try to teach them to assess the validity of their solutions, and to present (and explain) their solutions clearly enough so their colleagues and teachers will understand.

Initially, many students, especially the less confident ones, feel very insecure. Many feel like a student from one of my sections, who decided to talk to the chair of the department after first week in the course. She told him how frustrated she felt in this course. "This is not how I expected to be taught", she said. Happily, by the end of the semester, she was totally converted.

Students, who complete this course successfully, generally do well in follow-up courses. This is remarkable, since we don't cover as much theory as in many traditional "remedial" courses - we review factoring and prime numbers, fractions, percent, solving equations and inequalities, definition of functions and graphing.

I have been observing students taking our Precalculus and Mathematics for Elementary School Teachers courses after completing FAN X99 course at SFU. They appear more confident, more ready to work on problems, and more likely to actively participate in class discussions than students who have not been through FAN X99. I talked to several former FAN X99 students who were successful in our Math 100, Precalculus, course to find out what they believed was the most important or most helpful thing they had learned in FAN X99. The most common answer I heard was: "Learning to ask questions."

A few examples of what our students work on, with their "code names":

1. **Handshake Problem:** There are 32 people in class today. If everybody shakes hands with everybody else, what is the total number of handshakes?

2. **Locker Problem:** CF Gauss Senior Secondary had exactly 1000 students. Their lockers, numbered 1 to 1000, lined the main corridor of the school. One day, the students decided to perform the following number theory experiment, using the lockers. The first student opened all the 1000 lockers. The second closed every second locker, starting with the locker #2. The third student visited every third locker, starting with #3, closing all the lockers that were open, and opening all the lockers that were closed. The fourth student started with locker #4, and changed every fourth by closing all the lockers that were open, and opening all the lockers that were closed. They continued in this manner until all students had performed their task: the n-th student started with locker #n, and visited every n-th locker, closing the open ones and opening the closed ones. Which lockers were open after the students finished the experiment?
3. **Darts Problem:** My friend Peter has built a new dartboard for his son. The board has two regions: the centre circle, valued at 9 points, and the outside circle, valued at 4 points. What is the largest number that cannot be achieved as a score in this game? (Assume that you can continue the game as long as you wish, and that you can stop whenever you wish.)

4. **1001 Pennies Problem:** One thousand and one pennies are arranged in a row on a table. Every second coin is replaced with a nickel. Then every third coin is replaced with a dime. Finally, every fourth coin is replaced with a quarter. What is the total value of coins left on the table?

What we are trying to teach through these problems:

Handshake Problem:
We ask students to come with more than one way of solving this problem. Those who don't know how to start are advised to "play it out" at their table and then try to generalize.
Some students start by noticing that, since you don't shake hands with yourself, everybody shakes hands with 31 other people, and give $31 \cdot 32$ as their answer. By "playing it out" at their table, they usually realize that they double-counted, and that they need to divide by 2.
Others start by "playing it out": imagining that the whole class lines up, and shake hands in turn. First person shakes 31 hands, second 30 hands, etc., and the total is the sum $31 + 30 + 29 + \ldots\ 2 + 1$. Then we discuss whether the two ways of

calculating give the same answer and why. Using formulas without deriving them is not allowed here, so we derive a formula for summing numbers from 1 to n.

Locker Problem:
This problem is a great one to show that, if you don't know what to do, the best way is a systematic investigation: list the first 30 numbers or so, and "play it out", looking for a pattern. When you start seeing the pattern, formulate a hypothesis (all square numbers). Then, there are two ways of proving the hypothesis: either repeat the process for all 1000 numbers, or find a reason. We observe that the lockers are visited by the students whose numbers are factors of the locker number, that the last student to visit a given locker is the student with that number, and that, for the locker to stay open, we need an odd number of visits. Then the students are asked to list all factors of the first 30 numbers, and compare the results with their first activity. They quickly observe that square numbers are exactly the numbers with an odd number of factors. Several other activities related to factors and prime factors follow.

Darts Problem:
Another example where a systematic investigation is the best approach. After a discussion whether it is possible to find the largest number that cannot be a score, students are encouraged to list numbers from 0 to 30, and, for each of them, verify whether it can be a score in the game or not. Lots of mistakes in calculations here, but eventually 23 emerges as a possible candidate, and a hypothesis is formulated. We observe that numbers 24, 25, … 30 are possible scores. For each of then in turn, we look as which scores would follow if we keep scoring 4, and observe that the numbers can be classified as multiples of 4, 1 more than a multiple of 4, 2 more than a multiple of 4, and 3 more than a multiple of 4, and, therefore, that, if four consecutive numbers are possible scores, all numbers from there on are possible scores. (This takes time!)   Follow-up discussion: what happens if we change the scoring system, and choose a different pair of numbers. Will we always find the largest number that cannot be a score?

1001 Pennies Problem:
This is a great problem with which to begin discussing patterns, multiples, and yes, the need for systematic investigation. We again suggest starting by looking at the first 20 or 30 coins, and looking for a pattern. Common mistakes: assuming that every set of 4 coins will have the same value; or taking 10 or 20 as the length of the pattern. After realising that the minimal segment they need to consider is 12 coins (least common multiple), and therefore that there are 83 full repetitions of the pattern and a bit more, another mistake is possible: multiplying the value of the first 12 coins by the decimal you get from dividing 1001 by 12, rather than calculating the value of the last 5 coins separately. (The answer is $99.19)

From time to time, we have a student who draws the whole 1001 coins and calculates the total, sometimes even correctly.